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EFFECT OF TWO TEMPERATURE ON THE TIME HARMONIC BEHAVIOUR OF AN AXISYMMETRIC PROBLEM IN TRANSVERSELY ISOTROPIC THERMOELASTIC SOLID WITH GREEN-NAGHDI THEORY OF TYPE-II

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Abstract :

The present work is aimed at the thermoelastic interactions in a two dimensional homogeneous, transversely isotropic thermoelastic solids with two temperatures in the context of Green - Naghdi model of type-II due to time harmonic sources. The Hankel transform has been employed to find the general solution to the field equations. Concentrated normal force , normal force over the circular region and concentrated thermal source and thermal source over the circular region have been taken to illustrate the application of the approach. The components of displacements, stresses and conductive temperature distribution are obtained in the transformed domain. The resulting quantities are obtained in the physical domain by using numerical inversion technique. Numerically simulated results are depicted graphically. A comparison is made by showing the effect of two temperature, one temperature and anisotropy on the components of normal displacement, normal stress, tangential stress and conductive temperature.

Key words: Transversely isotropic, thermoelastic, time harmonic sources, Hankel transform, concentrated and distributed sources.

1.Introduction:

During the past few decades , widespread attention has been given to thermoelasticity theories that admit a finite speed for the propagation of thermal signals. In contrast to the conventional theories based on parabolic-type heat equation , these theories are referred to as generalized theories. Thermoelasticity with two temperatures is one of the non classical theories of thermomechanics of elastic solids. The main difference of this theory with respect to the classical one is a thermal dependence. Green and Naghdi(1991,1992,1993) developed three models for generalized thermoelasticity of homogeneous isotropic materials, which are labelled as models I, II, and III. The nature of these theories is such that when the respective theories are linearised , Model I reduces to the classical heat conduction theory(based on Fourier's law). The linearised version of model II and III permits propagation of waves at finite speed. Model II, in particular exhibits a feature that is not present in other established thermoelastic models as it does not sustain dissipation of thermal energy (Green and Naghdi 1993). In this model the constitutive equations are derived by starting with the reduced energy equation and by including the thermal displacement gradient among other constitutive variables.

Green-Naghdi's third model (GN-III) admits dissipation of energy. In this model the constitutive equations are derived by starting with the reduced energy equation, where the thermal displacement gradient in addition to the temperature gradient , are among the constitutive variables. This theory was pursued by many authors.. Chandrasekharaiah and Srinath (2000) discussed the thermoelastic waves without energy dissipation in an unbounded body with a spherical cavity.

Youssef(2006), constructed a new theory of generalized thermoelasticity by taking into account two-temperature generalized thermoelasticity theory for a homogeneous and isotropic body without energy dissipation and obtained the variational principle(Youssef 2013). Youssef et al. (2007) investigated State space approach of two temperature generalized thermoelasticity of infinite body with a spherical cavity subjected to different types of thermal loading. Chen and Gurtin (1968), Chen et al. (1968) and Chen et al. (1969) have formulated a theory of heat conduction in deformable bodies which depends upon two distinct temperatures, the conductive temperature φ and the thermo dynamical temperature T. For time independent situations, the difference between these two temperatures is proportional to the heat supply, and in absence of heat supply, the two temperatures are identical. For time dependent problems, the two temperatures are different, regardless of the presence of heat supply. The two temperatures T, φ and the strain are found to have representations in the form of a travelling wave plus a response, which occurs instantaneously throughout the body. Several researchers studied various problems involving two temperature.e.g. (Warren and Chen 1973;Quintanilla 2002; Youssef AI-Lehaibi 2007; Youssef AI -Harby 2007; Kaushal, Kumar and Miglani 2010; Kumar, Sharma and Garg 2014;Sharma and Marin2013;Sharma and Bhargav 2014; Sharma, Sharma and Bhargav 2013;Sharma and Kumar2013). The axisymmetric problems has been studied during the past decade by many authors.e.g. (Kumar and Pratap 2009; Sharma , Kumar and Ram2012 ;Kumar and Kansal 2013; Kumar,Kumar and Gourla2013). Inspite of these studies no attempt has been made to study the axisymmetric deformation in transversely isotropic medium with two temperature and without energy dissipation in frequency domain.

In the present investigation, a two dimensional axisymmetric problem in transversely isotropic thermoelastic solid without energy dissipation and with two temperature in frequency domain is investigated . The components of normal displacement, normal stress, tangential stress and conductive temperature subjected to concentrated normal force , normal force over the circular region and concentrated thermal source along with thermal source over the circular region are obtained by using Hankel transforms. Numerical computation is performed by using a numerical inversion technique and the resulting quantities are shown graphically.

2.Basic Equations

Following Youssef (2011)the constitutive relations and field equations in absence of body forces and heat sources are:

$$t_{ij} = C_{ijkl}e_{kl} - \beta_{ij}T \tag{1}$$

$$C_{ijkl}e_{kl,j} - \beta_{ij}T_{,j} = \rho\ddot{u}_i \tag{2}$$

$$K_{ij}\varphi_{,ij} = \beta_{ij}T_0\ddot{e}_{ij} + \rho C_E\ddot{T} \tag{3}$$

where

$$T = \varphi - a_{ij}\varphi_{,ij} \tag{4}$$

$$\beta_{ij} = C_{ijkl}\alpha_{ij} \tag{5}$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \quad i,j = 1,2,3 \qquad (6)$$

Here

$C_{ijkl}(C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk})$ are elastic parameters, β_{ij} is the thermal tensor, T is the thermodynamic temperature, T_0 is the reference temperature, t_{ij} are the components of stress tensor, e_{kl} are the components of strain tensor, u_i are the displacement components, ρ is the density, C_E is the specific heat, K_{ij} is the materialistic constant, a_{ij} are the two temperature parameters, α_{ij} is the coefficient of linear thermal expansion.

3. Formulation of the problem

We consider a homogeneous transversely isotropic , thermoelastic body initially at uniform temperature T_0. We take a cylindrical polar co-ordinate system (r, θ, z) with symmetry about z –axis. As the problem considered is plane axisymmetric, the field component $v = 0$, and u , w, and φ are independent of θ. We have used appropriate transformation following Slaughter(2002) on the set of equations (1)-(3) to derive the equations for transversely isotropic thermoelastic solid without energy dissipation and with two temperature and restrict our analysis to the two dimensional problem with $\vec{u} = (u, 0, w)$, we obtain

$$c_{11}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{1}{r^2}u\right) + c_{13}\left(\frac{\partial^2 w}{\partial r\partial z}\right) + c_{44}\frac{\partial^2 u}{\partial z^2} + c_{44}\left(\frac{\partial^2 w}{\partial r\partial z}\right) - \beta_1\frac{\partial}{\partial r}\left\{\varphi - a_1\left(\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r}\right) - a_3\frac{\partial^2 \varphi}{\partial z^2}\right\} = \rho\frac{\partial^2 u}{\partial t^2}$$

(7)

$$(c_{13} + c_{44})\left(\frac{\partial^2 u}{\partial r\partial z} + \frac{1}{r}\frac{\partial u}{\partial z}\right) + c_{44}\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) + c_{33}\frac{\partial^2 w}{\partial z^2} - \beta_3\frac{\partial}{\partial z}\left\{\varphi - a_1\left(\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r}\right) - a_3\frac{\partial^2 \varphi}{\partial z^2}\right\} = \rho\frac{\partial^2 w}{\partial t^2} \qquad (8)$$

$$K_1\left(\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r}\right) + K_3\frac{\partial^2 \varphi}{\partial z^2} = T_0\frac{\partial^2}{\partial t^2}\left(\beta_1\frac{\partial u}{\partial r} + \beta_3\frac{\partial w}{\partial z}\right) + \rho C_E\frac{\partial^2}{\partial t^2}\left\{\varphi - a_1\left(\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r}\right) - a_3\frac{\partial^2 \varphi}{\partial z^2}\right\}$$

(9)

Constitutive relations are

$$t_{rr} = c_{11}e_{rr} + c_{12}e_{\theta\theta} + c_{13}e_{zz} - \beta_1 T$$
$$t_{zr} = 2c_{44}e_{rz}$$
$$t_{zz} = c_{13}e_{rr} + c_{13}e_{\theta\theta} + c_{33}e_{zz} - \beta_3 T$$
$$t_{\theta\theta} = c_{12}e_{rr} + c_{11}e_{\theta\theta} + c_{13}e_{zz} - \beta_1 T$$

(10)

Where

$e_{rz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right)$, $e_{rr} = \frac{\partial u}{\partial r}$, $e_{\theta\theta} = \frac{u}{r}$, $e_{zz} = \frac{\partial w}{\partial z}$,

$T = \varphi - a_1\left(\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r}\right) - a_3\frac{\partial^2 \varphi}{\partial z^2}$

$\beta_{ij} = \beta_i \delta_{ij}$, $K_{ij} = K_i \delta_{ij}$

$\beta_1 = (c_{11} + c_{12})\alpha_1 + c_{13}\alpha_3$, $\beta_3 = {}_2c_{13}\alpha_1 + c_{33}\alpha_3$

Assume the time harmonic behaviour as $(u, w, \varphi)(r, z, t) = (u, w, \varphi)(r, z)e^{i\omega t}$

(11)

To facilitate the solution, the following dimensionless quantities are introduced

$r' = \frac{r}{L}$, $z' = \frac{z}{L}$, $t' = \frac{c_1}{L}t$, $u' = \frac{\rho c_1^2}{L\beta_1 T_0}u$, $w' = \frac{\rho c_1^2}{L\beta_1 T_0}w$, $T' = \frac{T}{T_0}$, $t'_{zr} = \frac{t_{zr}}{\beta_1 T_0}$

$t'_{zz} = \frac{t_{zz}}{\beta_1 T_0}$, $\varphi' = \frac{\varphi}{T_0}$, $a'_1 = \frac{a_1}{L}$, $a'_3 = \frac{a_3}{L}$ (12)

in equations (7)-(9) and after that suppressing the primes and using (11) and applying Hankel transforms defined by

$$\tilde{f}(\xi, z, \omega) = \int_0^\infty f(r, z, \omega) r J_n(r\xi)\, dr \tag{13}$$

on the resulting quantities, we obtain

$$\left(-\left(\xi^2 - \omega^2\right) + \delta_2 D^2\right)\tilde{u} - \xi\delta_1 D\tilde{w} + \left(\xi(1 - a_1\xi) - a_3\xi D^2\right)\tilde{\varphi} = 0$$

(14)

$$\delta_1\left(\frac{-\xi^2+1}{\xi}\right)D\tilde{u} + \left(\delta_3 D^2 - \left(\delta_2\xi^2 - \omega^2\right)\right)\tilde{w} - \left(\frac{\beta_3}{\beta_1}\left(\xi^2 a_1 + 1\right)D - D^3\right)\tilde{\varphi} = 0$$

(15)

$$(-\delta_4\omega^2\xi)\tilde{u} - \delta_5 D\tilde{w} + \left(\left(\frac{\delta_6 a_3}{L} + \frac{K_3}{K_1}\right)D^2 - \xi^2 - \delta_6\omega^2(1 + \xi^2)\right)\tilde{\varphi} = 0$$

(16)

Where $\delta_1 = \frac{c_{13}+c_{44}}{c_{11}}$, $\frac{c_{44}}{c_{11}} = \delta_2$, $\frac{c_{33}}{c_{11}} = \delta_3$, $\delta_4 = \frac{\beta_1^2 T_0}{K_1}$, $\delta_5 = -\frac{\beta_3\beta_1 T_0}{K_1}\omega^2$,

$\delta_6 = \frac{\rho C_E C_1^2}{K_1}$

After solution of the equations (14)-(16), using the radiation condition that $\tilde{u}, \tilde{w}, \tilde{\varphi} \to 0$ as $z \to \infty$, yields

$$\tilde{u} = A_1 e^{-\lambda_1 z} + A_2 e^{-\lambda_2 z} + A_3 e^{-\lambda_3 z} \tag{17}$$

$$\tilde{w} = d_1 A_1 e^{-\lambda_1 z} + d_2 A_2 e^{-\lambda_2 z} + d_3 A_3 e^{-\lambda_3 z} \tag{18}$$

$$\tilde{\varphi} = l_1 A_1 e^{-\lambda_1 z} + l_2 A_2 e^{-\lambda_2 z} + l_3 A_3 e^{-\lambda_3 z} \tag{19}$$

Where $\pm\lambda_i$ $(i = 1,2,3)$ are the roots of the equation

$$AD^6 + BD^4 + CD^2 + E = 0 \tag{20}$$

Where A , B, C,D, and E are listed in appendix A and the values of coupling constants d_i and l_i , are given in appendix B and A_i ,*i=1,2,3* being arbitrary constants.

4.Applications

Mechanical forces/ Thermal source acting on the surface

The boundary conditions are

$$(i)\ t_{zz}(r,z,t) = -P_1(r,t)$$

$$(ii) t_{zr}(r,z,t) = 0$$

$$(iii) \frac{\partial \varphi}{\partial z} = P_2(r,t) \tag{21}$$

$P_1(r,t)$, $P_2(r,t)$ are well behaved functions

Here $P_2(r,t)$ =0 corresponds to plane boundary subjected to normal force and $P_1(r,t)$ = 0 corresponds to plane boundary subjected to thermal point force.

Case 1. Concentrated normal force/ Thermal point source

When plane boundary is subjected to concentrated normal force/ thermal point force, then $P_1(r,t), P_2(r,t)$ take the form

$$(P_1(r,t), P_2(r,t)) = \left(\frac{P_1\delta(r)e^{i\omega t}}{2\pi r}, \frac{P_2\delta(r)e^{i\omega t}}{2\pi r}\right) \tag{22}$$

P_1 is the magnitude of the force applied , P_2 is the magnitude of the constant temperature applied on the boundary and $\delta(r)$ is the Dirac delta function.

Using the equations (10), (11) in the boundary conditions (21) and applying the transforms defined by(12) and (13) and substitute the values of $\tilde{u}$, $\tilde{w}$, $\tilde{\varphi}$ from (17)-(19) in the resulting equations, we obtain the expressions for the components of displacement, stress , and conductive temperature in case of concentrated normal force which are given in appendix C and in case of thermal point source are these are obtained by replacing Δ_i by Δ_i^* and P_1 with P_2, as listed in appendix D

Case II: Normal force over the circular region/ Thermal source over the circular region

Let a uniform pressure of total magnitude P_1 / thermal source of magnitude P_2 applied over a uniform circular region of radius *a* is obtained by setting

$$P_1(r,t), P_2(r,t) = \left(\frac{P_1}{\pi a^2} H(a-r)e^{i\omega t}, \frac{P_2}{\pi a^2} H(a-r)e^{i\omega t}\right) \tag{23}$$

Where $H(a-r)$ is the Heaviside unit step function.

Making use of dimensionless quantities defined by (11) and then applying Laplace and Hankel transforms defined by (12)-(13) on (23) ,we obtain

$$(\widetilde{P_1}(\xi,\omega), \widetilde{P_2}(\xi,\omega)) = \left(\frac{P_1}{\pi a}\frac{J_1(a\xi)}{\xi}, \frac{P_2}{\pi a}\frac{J_1(a\xi)}{\xi}\right)$$

The expressions for the components of displacements, stress and conductive temperature can be obtained by replacing $\frac{P_1}{2\pi}$ with $\frac{P_1}{\pi a}\frac{J_1(a\xi)}{\xi}$ and by replacing $\frac{P_2}{2\pi}$ with $\frac{P_2}{\pi a}\frac{J_1(a\xi)}{\xi}$ in equations (C.1)-(C.5) and in (D.1)-(D.5) respectively

5.Particular cases

(i) If $a_1 = a_3 = 0$, from equations (C.1) $-$ (C.5) and from (D.1) $-$ (D.5) we obtain the corresponding expressions for displacements, stresses and temperature change in thermoelastic medium without energy dissipation.

(ii) If we take $a_1 = a_3 = a$, $c_{11} = \lambda + 2\mu = c_{33}$, $c_{12} = c_{13} = \lambda$, $c_{44} = \mu$, $\beta_1 = \beta_3 = \beta, \alpha_1 = \alpha_3 = \alpha, K_1 = K_3 = K$ in equations (C.1) $-$ (C.5) and (D.1) $-$ (D.5) , we obtain the corresponding expressions for displacements, stresses and conductive temperature for isotropic thermoelastic solid with two temperature.

6. Inversion of the transforms

To obtain the solution of the problem in physical domain, we must invert the transforms in equations (26)-(30) These expressions are functions of $\xi \; and \; z$,and hence are of the form $\tilde{\hat{f}}(\xi, z, \omega)$. To get the function $f(r, z, \omega)$ in the physical domain, we invert the Hankel transform using

$$\hat{f}(r, z, \omega) = \int_0^\infty \xi f(\xi, z, \omega) J_n(\xi r) d\xi \quad (24)$$

The last step is to calculate the integral in equation (24). The method for evaluating this integral is described in Press et al. (1986). It involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

7.Numerical results and discussion

Copper material is chosen for the purpose of numerical calculation which is transversely isotropic. Physical data for a single crystal of copper is given by

$c_{11} = 18.78 \times 10^{10}\, Kgm^{-1}s^{-2}$, $c_{12} = 8.76 \times 10^{10}\, Kgm^{-1}s^{-2}$, $c_{13} = 8.0 \times 10^{10}\, Kgm^{-1}s^{-2}$
$c_{33} = 17.2 \times 10^{10}\, Kgm^{-1}s^{-2}$, $c_{44} = 5.06 \times 10^{10}\, Kgm^{-1}s^{-2}$ $C_E = 0.6331 \times 10^3 JKg^{-1}K^{-1}$
$\alpha_1 = 2.98 \times 10^{-5} K^{-1}$, $\alpha_3 = 2.4 \times 10^{-5} K^{-1}$, $\rho = 8.954 \times 10^3 Kgm^{-3}$,
$K_1 = 0.04 \times 10^2 Nsec^{-2}deg^{-1}$, $K_3 = 0.02 \times 10^2 Nsec^{-2}deg^{-1}$ Following Dhaliwal and Singh(1980), magnesium crystal is chosen for the purpose of numerical calculation(isotropic solid). In case of magnesium crystal like material for numerical calculations, the physical constants used are

$\lambda = 2.17 \times 10^{10} Nm^2$, $\mu = 3.278 \times 10^{10} Nm^2$, $K = 0.02 \times 10^2 Nsec^{-2}deg^{-1}$

$\omega_1 = 3.58 \times 10^{11} S^{-1}$ $\beta = 2.68 \times 10^6\; Nm^{-2}deg^{-1}$, $\rho = 1.74 \times 10^3 Kgm^{-3}$

$T_0 = 298K$, $C_E = 1.04 \times 10^3 Jkg^{-1}deg^{-1}$

The values of normal displacement w, normal force stress t_{zz} , tangential stress t_{zr} and conductive temperature φ for a transversely isotropic thermoelastic solid with two temperature (TIT) , isotropic thermoelastic solid with two temperature(IT) , and thermoelastic solid without two temperature (TWT) are presented graphically to show the impact of two temperature and anisotropy. The frequency parameter is taken as $\omega = .75$.

i). The solid line corresponds to (IT) for $a_1 = .02 = a_3$ ii) small dashed line corresponds to(IT) for $a_1 = .06 = a_3$
iii) solid line with centre symbol circle corresponds to (TWT) for $a_1 = a_3 = 0$
iv) small dashed line with centre symbol diamond corresponds to (TIT) for $a_1 = a_3 = .06$

7.1 Normal force on the boundary of the half-space

Case I: Concentrated normal force

Fig.1 shows the variations of normal displacement *w* with distance r . The values of *w* (IT) are increasing with a sharp increase in the initial range ,also values for) for $a_1 = .02 = a_3$ are greater than *w* (IT) for $a_1 = .06 = a_3$ whereas the trend of variations is ascending oscillatory for (TWT) and (TIT) Fig.2 exhibits the variations of normal stress t_{zz}. It is observed that values of t_{zz} (IT) are decreasing corresponding to both the parameters i.e. for $a_1 = .02 = a_3$ and $a_1 = .06 = a_3$with a sharp decrease near the loading surface , however the variations follow oscillatory pattern in case of (TIT) and (TWT). Fig.3 shows that variations in t_{zr} are in oscillatory form with difference in magnitude for all the four cases ,whereas the behaviour of (IT) ($a_1 = .06 = a_3$) and (TIT) is opposite oscillatory. Fig.4 interprets the behaviour of conductive temperature φ. Near the loading surface there is a sharp decrease in values of φ for (IT) (both cases) ,but away from the loading surface ,the pattern is oscillatory, however for (TIT) and (TWT) the variations are oscillatory in the whole range.

Case II: Normal force over the circular region

The trend of variations of normal displacement *w*, normal stress t_{zz}, tangential stress t_{zr} and conductive temperature φ for normal force over the circular region is similar to concentrated normal force with difference in their magnitude. At a first look it seems as mirror image of one another i.e. pattern is similar but the corresponding values are different. These variations are shown in figs. (5-8)

7.2 Thermal source on the boundary of half-space

Fig.9 shows the variations of normal displacement *w* with distance r . The values of *w* (IT) are decreasing with a sharp decrease near the loading surface and also values of *w* (IT) for $a_1 = .06 = a_3$ are greater than *w* (IT) for $a_1 = .02 = a_3$ whereas the trend of variations is ascending oscillatory for (TWT) and (TIT) Fig.10 exhibits the variations of normal stress t_{zz}. It is observed that values of t_{zz} for (IT)($a_1 = .06 = a_3$) and (TIT)($a_1 = .03, a_3 = .06$)are sharply decreasing in the range $0 \leq x \leq 2$ and follow oscillatory pattern afterwards , also small variations near zero are observed for (IT)($a_1 = .02 = a_3$) whereas the pattern is oscillatory in the whole range for (TWT) Fig.11 shows that variations in t_{zr} are in oscillatory form away from the loading surface for (IT) ($a_1 = .06 = a_3$) and (TIT)($a_1 = .03, a_3 = .06$)whereas near the loading surface these have opposite behaviour. Also variations in t_{zr} , for (IT) ($a_1 = .02 = a_3$) and (TWT) follow opposite behaviour in the initial range and have small variations near zero in the rest of the range. Fig.12. interprets the behaviour of conductive temperature φ. Near the loading surface there is a sharp decrease in values of φ for (TIT) ,but away from the loading surface ,the pattern is oscillatory, however the variations for (IT)(both cases) and (TWT) are oscillatory in the whole range.

Case-I: Thermal point source
Case-II: Thermal source over the circular region

The trend of variations of normal displacement *w*, normal stress t_{zz}, tangential stress t_{zr} and conductive temperature φ for thermal source over the circular region is similar to thermal point source with difference in their magnitude. The pattern is similar but the corresponding values are different. These variations are shown in figs. (12-16)

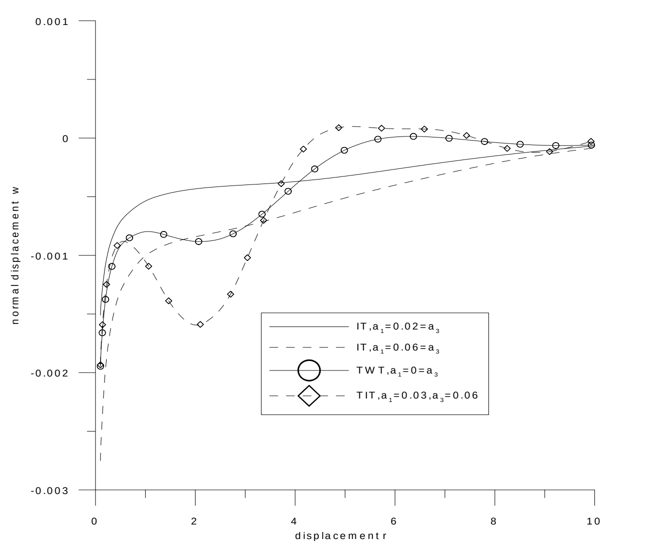

Fig.1.Variation of normal displacement w with distance r(concentrated normal force)

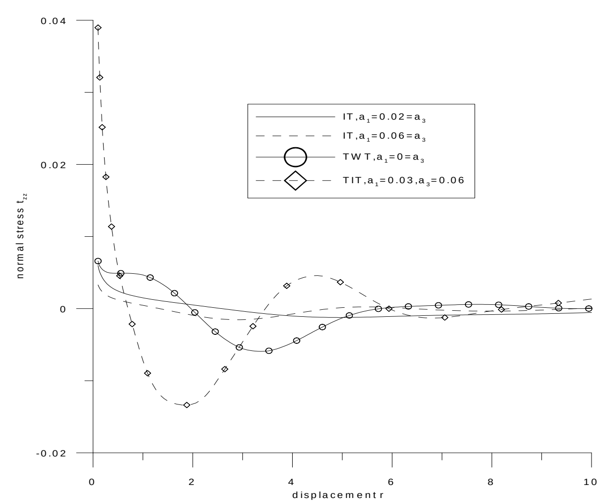

Fig.2.Variation of normal stress t_{zz} with distance r(concentrated normal force)

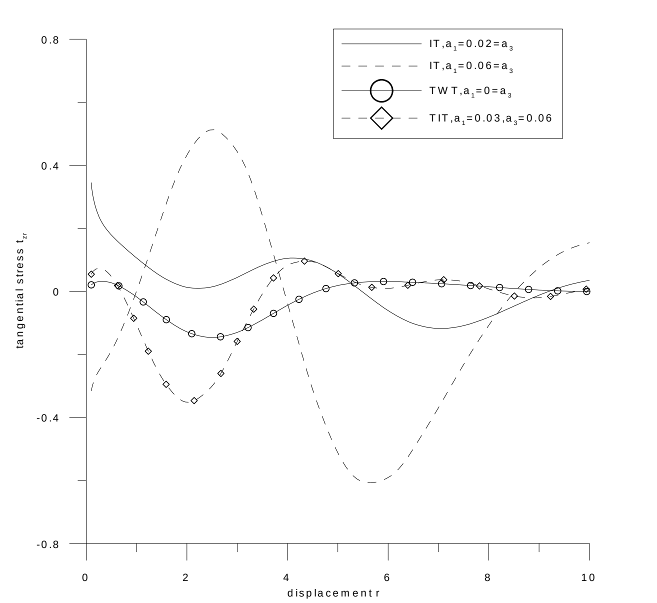

Fig.3. Variation of tangential stress t_{zr} with distance r(concentrated normal force)

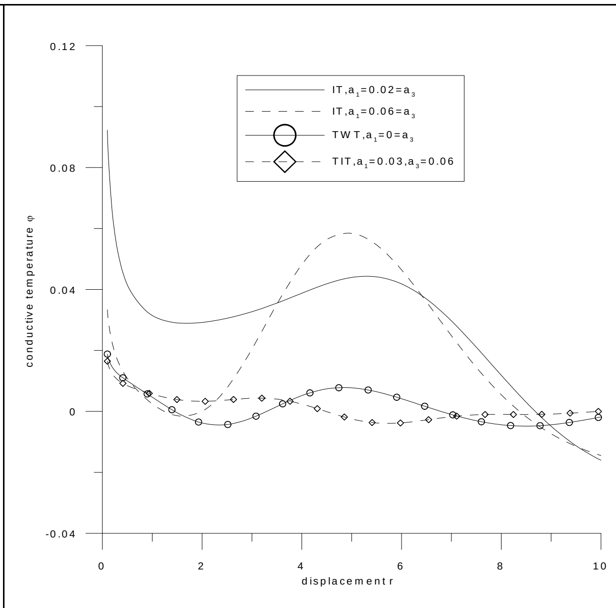

Fig.4. Variation of conductive temperature φ with distance r(concentrated normal force)

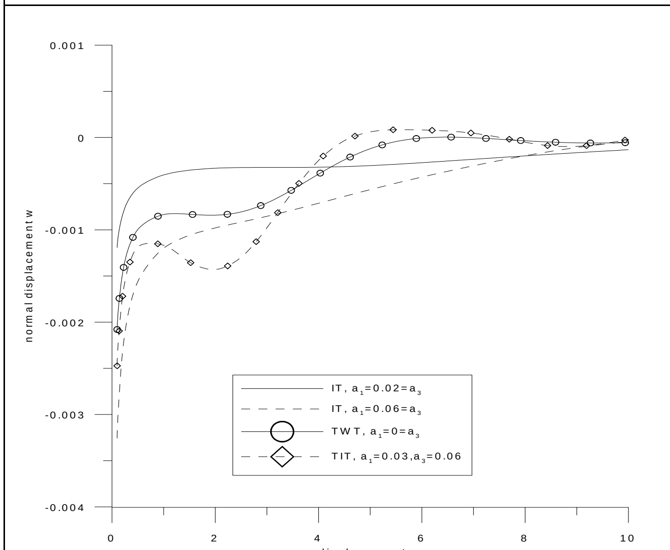

Fig.5.Variation of normal displacement w with distance r (normal force over the circular region)

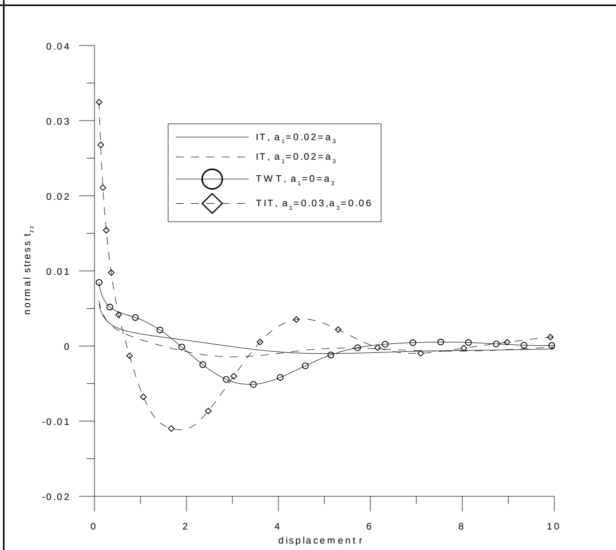

Fig.6. Variation of normal stress t_{zz} with distance r (normal force over the circular region)

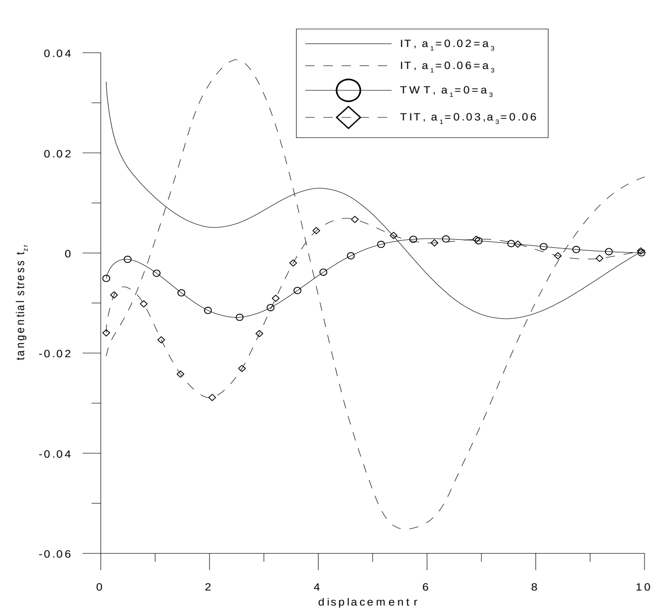

Fig.7. Variation of tangential stress t_{zr} with distance r (normal force over the circular region)

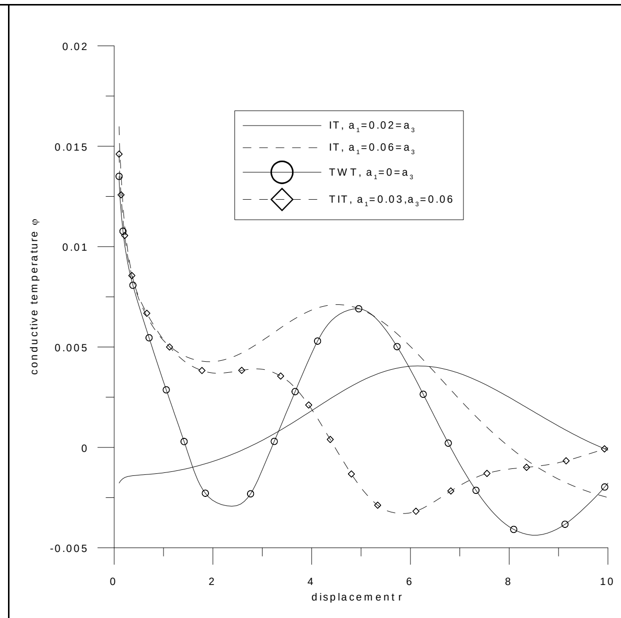

Fig.8. Variation of conductive temperature φ with distance r (normal force over the circular region)

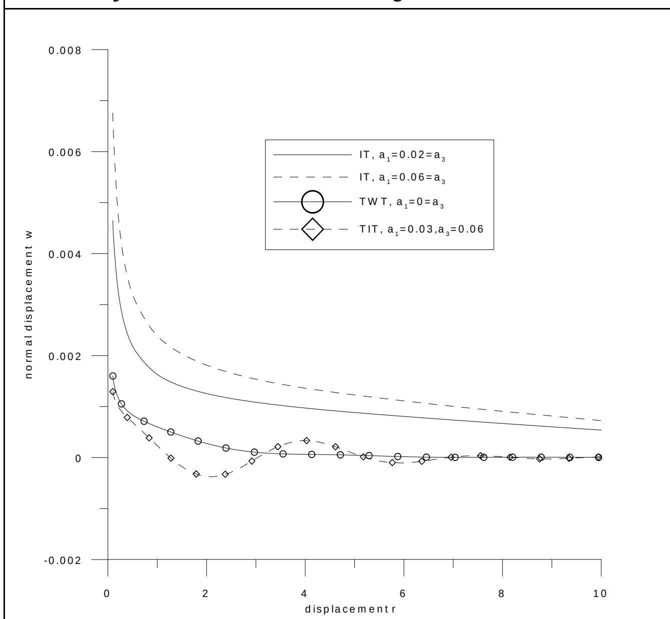

Fig.9 Variation of normal displacement w with distance r (thermal point source)

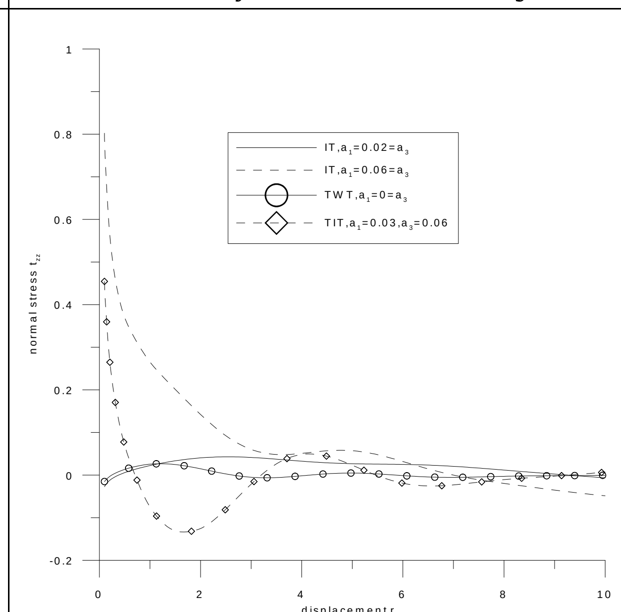

Fig.10 Variation of normal stress t_{zz} with distance r (thermal point source)

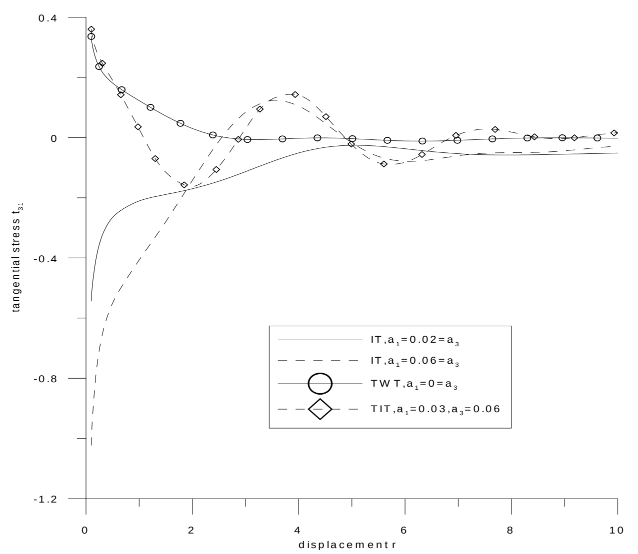

Fig.11 Variation of tangential stress t_{zr} with distance r (thermal point source)

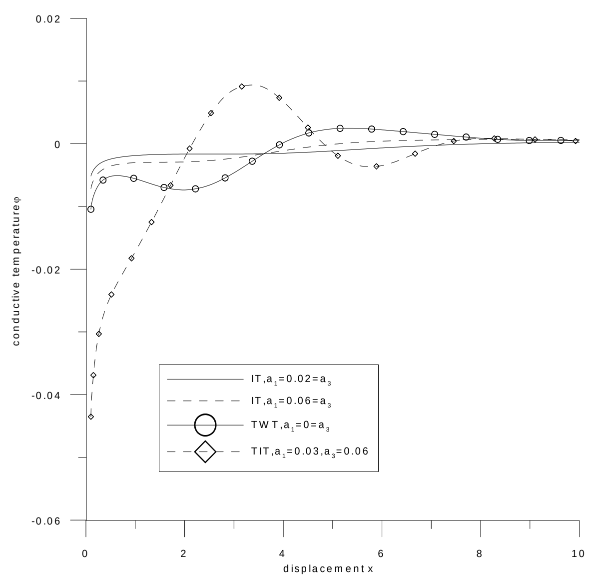

Fig.12 Variation of conductive temperature φ with distance r (thermal point source)

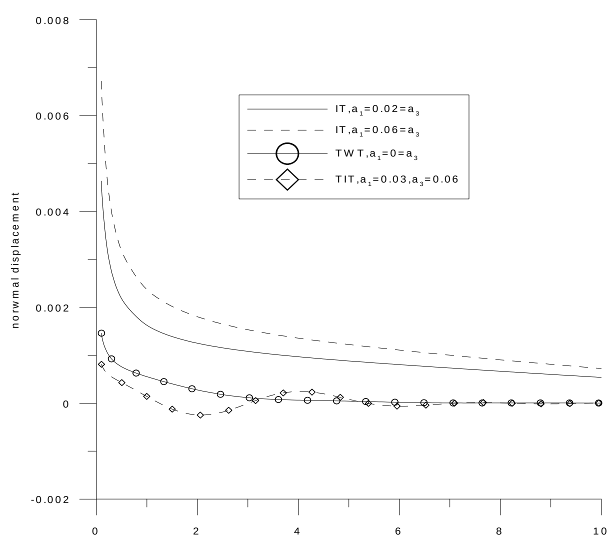

Fig.13 Variation of normal displacement w with distance r (thermal source over the circular region)

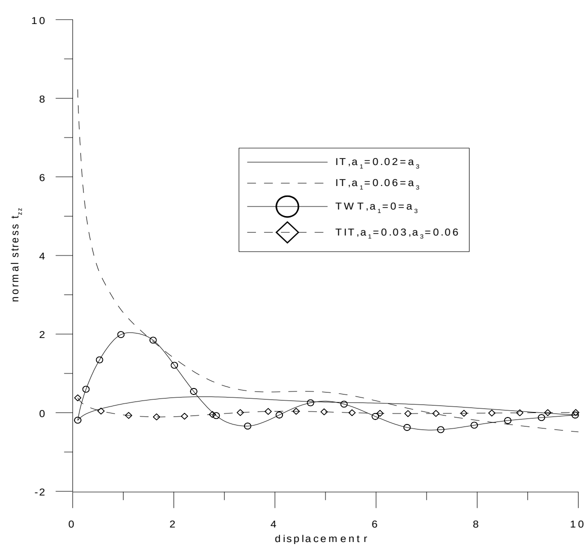

Fig.14. Variation of normal stress t_{zz} with distance r(thermal source over the circular region)

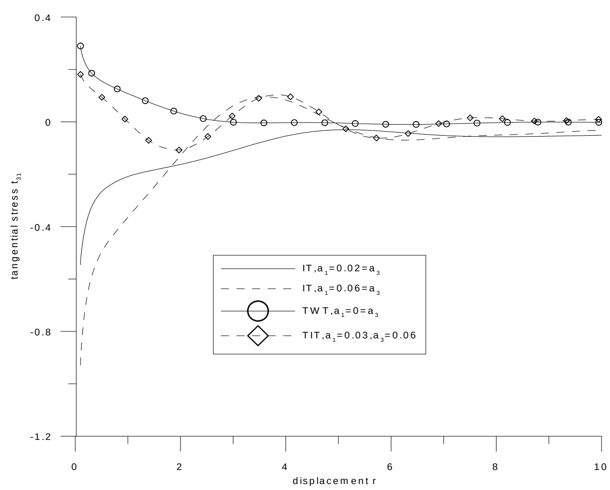

Fig.15. Variation of tangential stress t_{zr} with distance r (thermal source over the circular region)

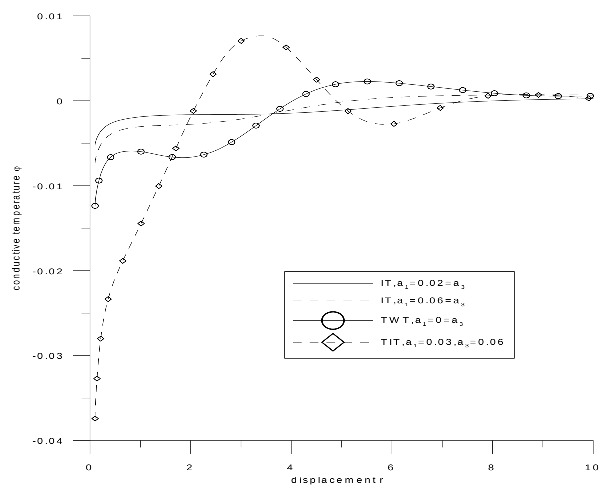

Fig.16. Variation of conductive temperature φ with distance r (thermal source over the circular region)

8. conclusion

From the graphs it is clear that effect of two temperature plays an important part in the study of the deformation of the body. Changing the two temperature parameter has a significant impact in the deformation of the isotropic body as is observed in the graphs. As r diverse from the point of application of the source the components of normal stress, tangential stress and conductive temperature for all types of sources (concentrated normal force / normal force over the circular region/ thermal point source/ thermal source over the circular region) follow an oscillatory pattern. It is observed that the variations of normal stress t_{zz}, tangential stress t_{zr} and conductive temperature φ for both mechanical forces (concentrated normal force and normal force over the circular region) are same and for both thermal sources(thermal point source and thermal source over the circular region) are same with difference in magnitude. As the disturbances travel through different constituents of the medium , it suffers sudden changes ,resulting in an inconsistent/ non- uniform pattern of curves. The results of this problem are very useful in the two dimensional problem of dynamic response due to various sources of the transversely isotropic thermoelastic solid with two temperature which has various geophysical and industrial applications. The problem of rotation disks or cylinders has its applications in high-speed cameras, steam and gas turbines, planetary landings and in many other domains.

Appendix A

$$A = \delta_1^2 \zeta_1 - K_1 \delta_5 \delta_1$$

$$B = -\zeta_2 \zeta_1 \delta_1 + \delta_5 \zeta_2 \mathrm{K}_1 - {\delta_1}^2 \zeta_4 - \delta_1 \zeta_1 \zeta_3 + \xi {\delta_2}^2 \zeta_1 \zeta_5 - \mathrm{K}_1 \xi \zeta_7 \zeta_6 + \xi \delta_2 \delta_5 \zeta_5 \mathrm{a}_3 + \mathrm{a}_3 \zeta_7 \xi \delta_3$$

$C = \zeta_2\delta_1\zeta_4 + \zeta_1\zeta_2\zeta_3 + \zeta_8\delta_5\zeta_2 + \delta_1\zeta_3\zeta_4 - \delta_1\zeta_5\zeta_8 - \xi\delta_2{}^2\zeta_5\zeta_4 + \xi\delta_2\delta_5\zeta_5 -$
$\zeta_6\zeta_5\delta_5\delta_2 - \zeta_6\zeta_7\delta_3 - a_3\xi\zeta_7\zeta_3$

$D = \zeta_6\zeta_3\zeta_7$

Where

$\zeta_1 = \left(\frac{\delta_6 a_3}{L} + \frac{K_3}{K_1}\right)$, $\zeta_2 = \xi^2 - \omega^2$ $\quad \zeta_3 = \delta_1\xi^2 - \omega^2$ $\quad \zeta_4 = \xi^2 - \delta_6\omega^2(1+\xi^2)$

$\zeta_5 = \frac{-\xi^2+1}{\xi}$ $\quad \zeta_6 = \xi(1 - a_1\xi), \zeta_7 = -\delta_4\xi\omega^2$ $\quad \zeta_8 = \frac{\beta_3}{\beta_1}(1 + a_1\xi^2)i\omega$

Appendix B

$d_i = \frac{-\lambda_i^3 P^* - \lambda_i Q^*}{\lambda_1^4 R^* + \lambda_1^2 S^* + T^*} \quad i = 1,2,3$

$l_i = \frac{\lambda_i^2 P^{**} + Q^{**}}{\lambda_1^4 R^* + \lambda_1^2 S^* + T^*} \quad i = 1,2,3$

Where $P^* = \delta_2\zeta_5\zeta_1 - \zeta_7 K_1$

$Q^* = \zeta_7\zeta_8 - \delta_2\zeta_4\zeta_5$

$R^* = \delta_3\zeta_1 - K_1\delta_5$

$S^* = \delta_5\zeta_8 - \zeta_3\zeta_1 - \zeta_4\delta_3$

$T^* = \zeta_4\zeta_3$

$P^{**} = -(\zeta_5\delta_2\delta_5 + \zeta_7\delta_3)$

$Q^{**} = \zeta_7\zeta_3$

Appendix C

$\tilde{u} = \frac{1}{\Delta}\left\{-\frac{P_1}{2\pi}(\Delta_1 e^{-\lambda_1 z} - \Delta_2 e^{-\lambda_2 z} + \Delta_3 e^{-\lambda_3 z}\right\} e^{i\omega t}$ (C.1)

$\tilde{w} = \frac{1}{\Delta}\left\{-\frac{P_1}{2\pi}(d_1 \Delta_1 e^{-\lambda_1 z} - d_2\Delta_2 e^{-\lambda_2 z} + d_3\Delta_3 e^{-\lambda_3 z}\right\} e^{i\omega t}$
(C.2)

$\widetilde{t_{zz}} = \frac{1}{\Delta}\left\{-\frac{P_1}{2\pi}(h_1 \Delta_1 e^{-\lambda_1 z} - h_2\Delta_2 e^{-\lambda_2 z} + h_3\Delta_3 e^{-\lambda_3 z}\right\} e^{i\omega t}$
(C.3)

$\widetilde{t_{zr}} = \frac{1}{\Delta}\left\{\frac{P_1}{2\pi}(m_1 \Delta_1 e^{-\lambda_1 z} - m_2\Delta_2 e^{-\lambda_2 z} + m_3\Delta_3 e^{-\lambda_3 z}\right\} e^{i\omega t}$
(C.4)

$\tilde{\varphi} = \frac{1}{\Delta}\left\{-\frac{P_1}{2\pi}(l_1 \Delta_1 e^{-\lambda_1 z} - l_2\Delta_2 e^{-\lambda_2 z} + l_3\Delta_3 e^{-\lambda_3 z}\right\} e^{i\omega t}$ (C.5)

Where

$\Delta_1 = (l_3\lambda_3)(\lambda_2 + \xi d_2) - (l_2\lambda_2)(\lambda_3 + \xi d_3)$

$\Delta_2 = (l_3\lambda_3)(\lambda_1 + \xi d_1) - (l_1\lambda_1)(\lambda_3 + \xi d_3)$

$\Delta_3 = (l_2\lambda_2)(\lambda_1 + \xi d_1) - (l_1\lambda_1)(\lambda_2 + \xi d_2)$

$\Delta = h_1\Delta_1 - h_2\Delta_2 + h_3\Delta_3$

$h_j = -\xi \frac{c_{31}}{\rho c_1^2} - \frac{c_{33}}{\rho c_1^2} d_j\lambda_j - \frac{\beta_3}{\beta_1} l_j + \frac{\beta_3}{\beta_1} l_j\lambda_j^2 a_3 - \frac{\beta_3}{\beta_1} l_j a_1 \xi^2, j = 1,2,3.$

$m_j = c_{44}\frac{\beta_1 T_0}{\rho c_1^2}(\lambda_j + \xi d_j) \quad j = 1,2,3$

Appendix D

$$\tilde{u} = \frac{1}{\Delta}\left\{-\frac{P_2}{2\pi}(\Delta_1^* e^{-\lambda_1 z} - \Delta_2^* e^{-\lambda_2 z} + \Delta_3^* e^{-\lambda_3 z}\right\} e^{i\omega t} \quad \text{(D.1)}$$

$$\tilde{w} = \frac{1}{\Delta}\left\{-\frac{P_2}{2\pi}(d_1\Delta_1^* e^{-\lambda_1 z} - d_2\Delta_2^* e^{-\lambda_2 z} + d_3\Delta_3^* e^{-\lambda_3 z}\right\} e^{i\omega t} \quad \text{(D.2)}$$

$$\tilde{t}_{zz} = \frac{1}{\Delta}\left\{-\frac{P_2}{2\pi}(h_1\Delta_1^* e^{-\lambda_1 z} - h_2\Delta_2^* e^{-\lambda_2 z} + h_3\Delta_3^* e^{-\lambda_3 z}\right\} e^{i\omega t} \quad \text{(D.3)}$$

$$\tilde{t}_{zr} = \frac{1}{\Delta}\left\{\frac{P_2}{2\pi}(m_1\Delta_1^* e^{-\lambda_1 z} - m_2\Delta_2^* e^{-\lambda_2 z} + m_3\Delta_3^* e^{-\lambda_3 z}\right\} e^{i\omega t} \quad \text{(D.4)}$$

$$\tilde{\varphi} = \frac{1}{\Delta}\left\{-\frac{P_2}{2\pi}(l_1\Delta_1^* e^{-\lambda_1 z} - l_2\Delta_2^* e^{-\lambda_2 z} + l_3\Delta_3^* e^{-\lambda_3 z}\right\} e^{i\omega t} \quad \text{(D.5)}$$

$\Delta_1^* = -(h_2)(\lambda_3 + \xi d_3) + (h_3)(\lambda_2 + \xi d_2)$

$\Delta_2^* = -(h_1)(\lambda_3 + \xi d_3) + (h_3)(\lambda_1 + \xi d_1)$

$\Delta_3^* = -(h_1)(\lambda_2 + \xi d_2) + (h_2)(\lambda_1 + \xi d_1)$

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